On an equivalence theorem and its applications to partial differential equations

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It is well known that a fundamental lemma due to J.L. Lions plays a fundamental role in the existence theory of some important partial differential equations of mathematical physics, such as those that model solid and fluid mechanics. However, each known proof of this lemma, in the realistic case of a domain with a boundary that is only Lipschitz-continuous, is notoriously difficult. In this talk, we will establish an "equivalence theorem", which shows that this lemma of J.L. Lions is in effect equivalent to several other fundamental results, such as an inequality due to J. Necas, or the surjectivity of the divergence operator between specific function spaces. A noticeable feature of this equivalence theorem is that its proof only requires a modicum of functional analysis. Since some of these other fundamental results results are amenable to a "direct" proof, such as the constructive proof of the surjectivity of the divergence operator due to M.E. Bogovskii, or that by J. Necas of the inequality that bears his name, any such "direct" proof thus provides a simple proof of J.L. Lions lemma by means of our equivalence theorem.

Reference: Chérif Amrouche, Philippe G. Ciarlet, Cristinel Mardare: On a lemma of Jacques-Louis Lions and its relation to other fundamental results, *Journal de Mathématiques Pures et Appliquées*, to appear in 2015.